Reality as Countermeasure? Statistical Risk Assessment of Passive Attacks on Encrypted Keyword Search

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- MSc. in CS from Univ. of Tech. of Compiègne 🚺, specialized in **Data Mining**.
- PhD student between Inria [] and University of Twente .
- Working on **privacy-preserving machine learning** under the supervision of Florian Hahn (UTwente), Andreas Peter (Uni. Oldenburg **=**), and Jan Ramon (Inria).
- Previously worked on **attacking searchable symmetric encryption**: Damie et al. (USENIX 2021), Dijkslag et al. (ACNS 2022).
- Still have a few SSE-related ideas in mind.



1. Introduction

2. The attacker knowledge, a noisy knowledge

3. Statistical risk assessment

4. Conclusion

Searchable Symmetric Encryption (SSE) 🔐



Attacks against SSE schemes 💥

- Similar-data attacks (based on co-occurrence information)
- Known-data attacks (based on co-occurrence information)
- Query-frequency attacks
- Active attacks
- Other attacks: against range queries, conjunctive-keyword search, etc.
- Our focus: similar-data attacks against static schemes with single-keyword search
- Our approaches can be extended to other settings.



What precisely does "similar" data mean? 🔍

• After our attack papers \Rightarrow **unsatisfied by the notion of "similar" data**.

• The ML literature is more specific regarding data distribution assumptions.

• We started exploring the limits of this similarity assumption using statistics.

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From statistical exploration to concrete SSE problems 📚

Our statistical exploration reached novel conclusions for two main problems:

Practicality of SSE attacks

All the attack papers successively improved state-of-the-art, but the literature gives **no tool** to evaluate their **efficiency in real-world scenarios**.

SSE attack analysis

The **parameters influencing attack accuracy** are unclear, and attack papers often make **arbitrary choices** in the experiments (e.g., uniform document set splitting).

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• A robust statistical method to **assess the risk** of deploying an SSE scheme in concrete use cases.

- We show that the uniform dataset splitting used in all attack papers simulates an **advantageous scenario for the attacker** (i.e., the best source of similar doc.).
- An **attack analysis methodology** based on a similarity metric. We provide several novel conclusions about the parameters influencing attack accuracy.

Paper under submission...

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• A robust statistical method to **assess the risk** of deploying an SSE scheme in concrete use cases. [**Focus of this presentation**]

- We show that the uniform dataset splitting used in all attack papers simulates an **advantageous scenario for the attacker** (i.e., the best source of similar doc.).
- An **attack analysis methodology** based on a similarity metric. We provide several novel conclusions about the parameters influencing attack accuracy.

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Simplified attacker knowledge: co-occurrence matrices 📝

Let n_{ind} (resp. n_{atk}) be the size of D_{ind} (resp. D_{atk}).



Revisiting the co-occurrence matrices 🔬

Our intuition

As in ML, we consider a **dataset as a sample of a random distribution**. We want to leverage the **randomness contained in the document sets**.

Co-occurence matrix distribution

The co-occurrence matrix is drawn from a random matrix distribution composed of (dependent) **Binomial variables**. Details in the paper.

NB: D_{ind} and D_{atk} can have different random distributions.

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Towards a statistical hardness assumption 🤍

A Estimation vs. probabilities: C_{query} and C_{atk} = estimators of unknown proba.

SSE attack as an estimation problem

- SSE attack problem \approx representative sampling for a survey.
- \Rightarrow attack success depends on the knowledge **size**, **quality and distribution**.

Statistical hardness assumption

- Classic crypto: **computationally expensive** cryptoanalysis \Rightarrow sec. guarantee.
- Encrypted search: **unlikelihood** of having a precise estimation (i.e., a "similar enough" dataset) ⇒ sec. guarantee.
- Risk assessment quantifies the statistical hardness.



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Concrete deployment problem

A company wants to deploy encrypted mailboxes with SSE for its employees.

Existing solutions to assess the risk?

- Consider the research results on Enron and Apache datasets ⇒ Problem: Enron and Apache are not similar (i.e., cannot represent all email use cases)
- The company has a dedicated sample dataset ⇒ <u>Problem</u>: the dataset size limits the simulations (e.g., cannot simulate attacks with large attacker knowledge).



Problems with theoretical bounds

- SSE attack problem is **complex**: \mathcal{NP} -complete, **dependent** random variables.
- A theoretical bound could be **non-informative** (i.e., too loose).
- Any scheme modification (e.g., attack mitigation) requires a new analysis.

Benefits of empirical bounds

- **Consider the use case specificities** (*via a sample dataset*) to obtain tight bounds.
- Support search scheme modifications, such as attack countermeasures.

 \Rightarrow **Our objective**: a method to **bound the attack accuracy** for a given use case (i.e., based on a **sample dataset representative** of the use case).

Estimating an empirical bound 📉



Figure: Accuracy upper bound of the IHOP attack (quantile: 0.95)



Conservative risk assessment

"Advantageous" simulation parameters: realistic attackers cannot benefit from better conditions.

Quantile regression

A quantile regression estimates (b, a) s.t. $Q_Y(\alpha) = b \cdot X + a^{\dagger} \Rightarrow$ ideal for a bound estimation.

Our upper bound function

 $Q_{Acc}(\alpha; n_{ind}, n_{atk}) = expit(b \cdot \log(\frac{1}{n_{ind}} + \frac{1}{n_{atk}}) + a).$ Detailed motivations in the paper.

[†] $Q_{Y}(\alpha)$: quantile α of data distribution Y.

Supporting real-world deployments 🛠

Setting a maximum index size

Deduce n_{\max} s.t. $\lim_{n_{\mathsf{atk}}\to\infty} Q_{\mathsf{Acc}}(\alpha; n_{\max}, n_{\mathsf{atk}}) < \mathsf{negl}$

Security guarantee

If the size limit is respected, the **attack accuracy remains negligible** with high probability.

Limitation

The estimated upper bound holds for a **specific attack** on a **given use case**.



Figure: Accuracy upper bound of the IHOP attack (quantile: 0.95)

Risk assessment pipeline 🔆

- Find a sample dataset representative of the use case.
- **Simulate attacks** using this dataset and the advantageous simulation parameters identified in the paper.
- Compute the **quantile regression** on the simulation results.
- Estimate a **maximum index size** and decide whether it is too low for the use case.

Reproduce this protocol if new attacks are released (or if the use case evolves).

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Tuning the security of SSE deployments 🔧

- Maximum index size could be too small ⇒ insecure use case by default.
- Solution: attack mitigation techniques.
- Risk assessment helps choose parameters **minimizing the overhead**.
- Can also tune the secure index parameters (e.g., queryable vocabulary).



Figure: Accuracy upper bound with varying mitigation parameters



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- The **stochastic model of co-occurrence matrices** provides a novel understanding of the SSE attack problem.
- We conceived a simple **risk assessment protocol based on robust statistical tools** to support real-world deployments.
- Some use cases can be deployed **securely without dedicated attack mitigation techniques**.
- We also provide various novel insights about attack analysis methodology.





- A unified security framework for all privacy-preserving technologies with statistical leakage (including SSE and PPML)? Bayes security measure [CSF'23]?
- **Formalizing the notion of statistical hardness** assumption.
- **Building upon recent papers**? (Gui et al. [2023], Kornaropoulos et al. [2022])
- Extending the risk assessment and similarity analysis to other settings: range queries, active attacks, query-frequency attacks, etc.

Contact me if you want to collaborate on these topics: marc.damie@inria.fr

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Thank you for your attention!

Additional slides Uniform document set splitting, a favored attacker simulation



Let C_{ind} be the matrix C_{query} with the same rotation as C_{atk} .

Definition

The document sets D_{ind} and D_{atk} are ϵ -similar if:

$$\mathbf{x} = \left| \left| \frac{C_{\mathsf{ind}}}{n_{\mathsf{ind}}} - \frac{C_{\mathsf{atk}}}{n_{\mathsf{atk}}} \right| \right|$$

Interpretation

The ϵ -similarity quantifies the divergence between two document sets.

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All attack papers use uniform splitting (e.g., on the Enron email dataset) to generate the document sets in their experiments.

Goal of this contribution

Shows that uniform splitting \Rightarrow best-case scenario for the simulated attacker.

Steps

- Uniform splitting contrary to other methods \Rightarrow equal document set distributions
- Equal (document set) random distributions \Rightarrow smaller ϵ -similarity
- Smaller ϵ -similarity \Rightarrow higher accuracy [Done in a previous paper]



Uniform sampling \Rightarrow equal document set distributions

Let p_{ind} and p_{atk} parametrize the random distributions of C_{ind} and C_{atk} .

Statistical test

We conceived a **statistical test** for the hypothesis $p_{ind} = p_{atk}$ ($p_{ind}, p_{atk} \in [0, 1]^{m \times m}$).

Experimental results

Tested the hypothesis with two sampling methods:

- Uniform sampling \Rightarrow Test not rejected (*p*-value always above 0.01).
- Year sampling \Rightarrow test strongly rejected (*p*-value below machine epsilon).



Equal random distributions \Rightarrow smaller ϵ -similarity

Let $\mathcal{E}^{p_{\mathrm{ind}},p_{\mathrm{atk}}}$ be the random distribution of the ϵ metric.

Stochastic Dominance

Let *X*, *Y* be two random distributions, $X \preccurlyeq Y \iff \forall z, \mathbb{P}(X \ge z) \le \mathbb{P}(Y \ge z)$

Our result

We prove that asymptotically: $\mathcal{E}^{p_{\text{ind}},p_{\text{ind}}} \preccurlyeq \mathcal{E}^{p_{\text{ind}},p_{\text{atk}}}$.

Interpretation

Equal document set distributions stochastically produce smaller ϵ

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Additional slides Attack analysis based on a similarity metric

Attack analysis based on a similarity metric 📊

Goal of this contribution

Use a similarity metric to improve attack analysis and comparison.

Example novel insight

The document set similarity is not the only factor influencing attack success.

Attack comparison

 $\epsilon\text{-similarity}$ + regression techniques \Rightarrow consistent and interpretable results.





Figure: Comparison of the estimation accuracy functions for three attacks.



A few novel insights about SSE attacks

• Indexed and attacker document set sizes have a **symmetric influence on accuracy**.

• Document set similarity is **not the only factor influencing attack success**.

• Leakage does not need to be indistinguishable, **just noisy enough**.

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